Double Penguins and the Contribution of Vector Meson–like States to the Decays $B \to K^*\gamma$, $B \to \rho\gamma$

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Using perturbative QCD, the contribution at the leading twist, leading α_s level, of charm and up quark loops to the decays $B \to K^*\gamma$ and $B \to \rho\gamma$ is presented. In the case of $B \to \rho\gamma$, the relative importance of these contributions depend upon the unknown CKM matrix elements V_{bu} and V_{td}. Assuming that the ratio $r = V_{bc}V^*_{cd}/V_{bt}V^*_{td}$ is bounded between $-2.25 \le r \le -.5$ as is suggested by the Particle Data Group, the error in extracting $|V_{td}/V_{ts}|$ by these decays is estimated.

I. INTRODUCTION

The recent observation [1] of the rare decay $B \to K^*\gamma$ was the first unambiguous experimental verification of flavor–changing neutral currents. While not occurring at tree–level in the Standard Model, such currents, or "penguins", are well–known [2] to arise due to loop effects. The magnitude of these transitions depend [3] upon fundamental parameters in the Standard Model such as the mass of the top quark as well as the CKM matrix elements $V_{tb}V^*_{ts}$, although significant QCD enhancements [4] tend to offset somewhat the dependence on the former. A systematic determination of the two–body rare decays of the B meson, both for determining these fundamental parameters as well as for indications of possible physics beyond the Standard Model, forms a major part of the goals of the future B factories at SLAC and KEK. Despite these connections, significant theoretical input will nevertheless be necessary to interpret the data so as to distinguish purely hadronic effects as well as potentially competing mechanisms.

With the latest data on $B \to J/\psi\, K^*$ [5], the question of the relative importance [6] of "long–distance" effects arising from intermediate charmonium states to the decay $B \to K^*\gamma$ has received new attention [7]. The potential relevance of annihilation diagrams in the decay $B \to \rho\gamma$ has also been emphasized by Atwood *et al.* [7] as a possible source of contamination of the extraction using these two decay modes [8] of the ratio $|V_{td}/V_{ts}|$. While these effects are undoubtedly present, it is crucial for their analysis that all mechanisms are understood within a single formalism. For example, most of the studies cited employ vector–meson dominance to estimate from the measured charmonium decay rate a contribution to the $B \to K^*\gamma$ transition amplitude purportedly distinct from the $b \to s\gamma$ penguin vertex already present in the effective Hamiltonian. However the (significant) running of the Wilson coefficient of the electromagnetic penguin operator in the effective Hamiltonian is predominantly driven by its mixing with other operators via charm–quark loops. There is hence a serious potential of double–counting when using a mixed meson–quark language to describe the transition amplitude. This potential is further emphasized when recalling that the mixing between O_7 and O_2 (see Eqs. (7) and (11) below) is in fact identically zero at the one–loop level. [4,9]

Recently, perturbative QCD (pQCD) methods were applied to the penguin decay $B \to K^*\gamma$ [10]. Previously pQCD had been found [11] to be quite successful [12] in describing the hadronic, two–body decay channels of the B meson. The large mass scale of the decaying B meson, coupled with the restriction to the two body exclusive decay modes involving nearly massless, highly Lorentz contracted states, implies that the transition amplitude is governed by short–distance processes (on hadronic scales) and hence an appropriate environment to apply pQCD. Indeed such simple kinematic considerations raises the question as to the true suitability of calling any mechanism contributing to these decays "long–distance" and further emphasizes the need for a unified approach.

The dominant graph in the pQCD framework that contributes to the decay $B \to K^*\gamma$ is shown in Fig. (1). A branching ratio of roughly 3×10^{-5} was obtained in Ref. [10], in reasonable agreement with the data [1]. One process that was however omitted in that analysis was the so–called double penguin graphs of Fig. (2). Based on the analysis of Ali and Greub [9] to the decay $b \to s\,\gamma\,g$, it was argued that such graphs would be suppressed compared to the dominant decay mechanism shown in Fig. (1). The fact that the virtuality of the gluon in Fig. (2) is in general significantly off–shell (of $O(m_c^2)$ in practice) and is embedded in additional loops, implies that the analytical expressions found in Ref. [9] using on–shell gluon emission could not be simply applied. A reasonable foundation for omitting the graph was therefore welcomed and exploited.

For a detailed understanding of the decay amplitude, and especially for the extrapolation to the case $B \to \rho\gamma$ where CKM factors no longer suppress other background processes, it is nevertheless important to quantify these additional mechanisms. This paper reports the results of such an analysis. While as we will see, these results do in fact support the conjecture that these mechanisms enhance the total decay rate, nontrivial hadronic phases enter into obtaining this result. This possibility

has been generally ignored. The enhancement is related to the observed [5] failure of factorization in the decay $B \to J/\psi K^*$, although the overall enhancement factor of the decay rate itself depends sensitively on details of the wavefunction of the K^* in a fashion that the dominant decay mechanism, Fig. (1), does not.

II. CALCULATIONS

Exclusive processes at large momentum transfer are addressed [13] within pQCD starting with a Fock component expansion of the involved hadrons whereby a twist expansion suggests that the contribution from the lowest order Fock component dominates the physical observable under consideration. An exclusive process then involves a perturbatively calculable hard amplitude convoluted with a nonperturbative, soft physics wavefunction, ψ_m, from each of the hadrons m entering or leaving the hard interaction. These wavefunctions, although as yet uncalculable from first principles, are universal for each meson, *i.e.* they factorize from the hard amplitude and hence are independent of the process involved. In exclusive processes in pQCD they play the analogous role that structure functions do in the case of inclusive scattering events. Thus as was employed in Ref. [11], ideally one can phenomenologically parametrize these wavefunctions using a (few) measured cross–sections/decay rates.

The factorization scheme advocated by Brodsky and Lepage [13] is employed, whereby the momenta of the quarks are taken as some fraction x of the total momentum of the parent meson weighted by a soft physics distribution amplitude $\phi(x)$. The peaking approximation is used for ϕ_B, the distribution amplitude of the B meson, wherein

$$\phi_B(x) = \frac{f_B}{2\sqrt{3}}\delta(x - \epsilon_B). \tag{1}$$

The decay constant of the B is f_B (in the convention $f_\pi = 93\text{MeV}$) and x is the light cone momentum fraction carried by the light quark. The parameter ϵ_B in $\phi_B(x)$ is related to the difference in the masses of the B meson and b-quark,

$$m_B = m_b + \bar{\Lambda}_B \tag{2}$$

whereby $\epsilon_B = \bar{\Lambda}_B/m_B$.

Ignoring the mass of the K^*, its distribution amplitude can be written as

$$\phi_{K^*}(y) = \sqrt{3} f_{K^*} y(1-y)\tilde{\phi}_{K^*}(y). \tag{3}$$

Two guesses for $\phi_{K^*}(y)$ were considered in [10],

$$\begin{aligned} \phi_{K^*}(y) &= 1 \\ \phi_{K^*}(y) &= 5\,y^2\,(1-y)^2. \end{aligned} \tag{4}$$

The first is the so–called super-asymptotic [14] distribution amplitude, while the lower form is one suggested by Chernyak, Zhitnitsky and Zhitnitsky (CZZ) [15] for the transverse polarizations of the ρ meson. *

In the present context, the factorization scheme is augmented by the viability of an ϵ_B expansion for the decay amplitude. All terms in the hard amplitude of order ϵ_B^2 are ignored, both because they are expected to be numerically small and because they are related, through the mass of the light quark, to transverse momentum effects. Since the latter is ignored in the factorization scheme, self–consistency dictates these other terms also be ignored. The parameter ϵ_B has been fitted [11,17] using the decay $B \to D\pi$. With mild assumptions concerning the decay constants f_B and f_D, a typical value found was $\epsilon_B = .095$.

A. Dominant mechanism

In Ref. [10], Fig. (1) was shown to dominate *all* graphs involving penguin operators. To leading order in ϵ_B, the contribution to the decay $B^+ \to K^{*+}\gamma$ was found to be

$$\begin{aligned} M_{\gamma\text{peng}} = &-\frac{8GV_{tb}V_{ts}^*}{m_B\epsilon_B}\alpha_s(\mu)C_7(\mu)I \\ &\times\left(p\cdot q\epsilon^*\cdot\xi^* + i\epsilon_{\mu\nu\alpha\beta}p^\mu q^\nu\epsilon^{*\alpha}\xi^{*\beta}\right), \end{aligned} \tag{5}$$

where ϵ (ξ) is the polarization of the photon (K^*),

$$G = \frac{eC_F}{4\pi}\frac{G_F}{\sqrt{2}}f_B f_{K^*}, \tag{6}$$

and $C_7(\mu)$ is the renormalization group improved [4] Wilson coefficient of the electromagnetic penguin operator

$$O_7 = \frac{e}{16\pi^2}\ m_b\bar{s}\sigma^{\mu\nu}F_{\mu\nu}\frac{1}{2}\left(1+\gamma_5\right)b. \tag{7}$$

The quantity I involves an integral over the distribution amplitude of the K^* and is given by

$$I = \int_0^1 dy\,\tilde{\phi}_{K^*}(y)\,\frac{(1-y)(1+y-2\epsilon_B)}{y - 2\epsilon_B - i0^+}. \tag{8}$$

For the two distribution functions considered, this becomes (using $\epsilon_B = .095$ as discussed earlier)

$$I = \begin{cases} .68 + i\,2.55 \\ 1.75 + i\,1.96 \end{cases}, \tag{9}$$

where the upper number is for the asymptotic distribution and the lower is for the CZZ one. The imaginary part comes from an internal propagator going on–shell,

*See however Ref. [8] for a QCD sum rule result for $\phi_{K^*}(y)$

kinematically allowed here by the inputed soft–physics parametrization Eq. (2) that $m_b < M_B$. As in other cases in pQCD [16], it is a calculable hadronic phase since the overall kinematics of the reaction dictate that only short distance propagation occurs, as discussed in [17]. In more technical language, the pole is not pinched and hence not associated with a long distance event [18].

One curious result of Eq. (9) is that I^2 is nearly identical despite the significant differences in phase for the two distribution amplitudes under consideration. Hence the total decay rate was found in [10] insensitive to these soft physics inputs. This pattern however does not continue when additional mechanisms are included in which interference effects depend crucially on the details of the relative phases.

B. The Annihilation graphs

The first such competing mechanism, the annihilation diagrams of Fig. (3) was already considered in [10]. [†] As suggested by the figure, the dominant result (in $1/\epsilon_B$) is from the graph involving photon emission off of the light–quark of the B meson. To this order in ϵ_B, the contribution of this process to the decay $B^+ \to K^{*+}\gamma$ is

$$\begin{aligned} M_{\rm ann} = &-\frac{2e_u\left(C_2(\tilde\mu)+\frac{1}{N_c}C_1(\tilde\mu)\right)}{m_B\epsilon_B}\frac{m_{K^*}}{m_B} \\ &\times\left[e\frac{G_F}{\sqrt2}V_{ub}V_{us}^* f_B f_{K^*}\right] \\ &\times\left(p\cdot q\epsilon^*\cdot\xi^* + i\epsilon_{\mu\nu\alpha\beta}p^\mu q^\nu\epsilon^{*\alpha}\xi^{*\beta}\right), \end{aligned} \tag{10}$$

where $e_u = 2/3$ and C_2 and C_1 are the Wilson coefficients of the 4–point operators (greek subscripts are color indices)

$$\begin{aligned} O_1 &= \frac14\bar u_\alpha\gamma^\mu(1-\gamma_5)b_\beta\,\bar s_\beta\gamma_\mu(1-\gamma_5)u_\alpha, \\ O_2 &= \frac14\bar u_\alpha\gamma^\mu(1-\gamma_5)b_\alpha\,\bar s_\beta\gamma_\mu(1-\gamma_5)u_\beta. \end{aligned} \tag{11}$$

In this case m_{K^*} is kept as it appears as an overall factor arising from the W turning into a K^*. Due to the differences in CKM matrix elements as well as the factor m_K^*/M_B, the annihilation amplitude is here essentially ignorable. As though discussed by Cheng and also Atwood *et al.* [7], this is no longer true for the (as yet unseen) decay mode $B \to \rho\,\gamma$. For the decay $B^+ \to \rho^+\,\gamma$, M_{ann} is obtained from Eq. (10) with the obvious modification in CKM factors and meson decay constant. For the neutral decay $B^0 \to \rho^0\,\gamma$,

$$\begin{aligned} M_{\rm ann} = &-\frac{2e_d\left(C_1(\tilde\mu)+\frac{1}{N_c}C_2(\tilde\mu)\right)}{m_B\epsilon_B}\frac{m_{K^*}}{m_B} \\ &\times\left[e\frac{G_F}{\sqrt2}V_{ub}V_{ud}^* f_B \frac{f_\rho}{\sqrt2}\right] \\ &\times\left(p\cdot q\epsilon^*\cdot\xi^* + i\epsilon_{\mu\nu\alpha\beta}p^\mu q^\nu\epsilon^{*\alpha}\xi^{*\beta}\right). \end{aligned} \tag{12}$$

C. The Quark-Loop graphs

We now come to the second competing mechanism, the quark–loop graphs of Fig. (2). These have been recently considered by Greub *et al.* in Ref. [20] using a quark model approach. These authors were predominantly interested in studying CP violating effects and thus focussed on only the absorptive parts of these graphs. Here though, the entire amplitude is of interest. The important application to CP violation will be deferred to a later work.

In the evaluation of the graphs of Fig. (2), one finds that while each graph is individually ultraviolet divergent, their sum is finite. Likewise gauge–invariance (in both the strong and electromagnetic interactions) is only obtained after the graphs are summed. In the case that the gluon is on–shell, $Q^2 = 0$, entirely analytical results [9] are possible. Such a convenient form has not been found in the present context which involves a second integration over the gluon's virtuality, $Q^2 = -y\epsilon_B M_B^2$. Intermediate results will be presented, in which the remaining integrals were then evaluated numerically. The contribution of the two quark loop graphs of Fig. (2), $M_{q\bar q\,{\rm loop}}$, to the decay amplitude is

$$\begin{aligned} M_{q\bar q\,{\rm loop}} = &-\frac{16GV_{qb}V_{qs}^*}{3m_B\epsilon_B}\alpha_s(\tilde\mu)C_2(\tilde\mu)\int_0^1 dy\,\tilde\phi_{K^*}(y)\,\tilde I(y) \\ &\times\left(p\cdot q\epsilon^*\cdot\xi^* + i\epsilon_{\mu\nu\alpha\beta}p^\mu q^\nu\epsilon^{*\alpha}\xi^{*\beta}\right), \end{aligned} \tag{13}$$

where $\tilde I(y)$ is

$$\begin{aligned} \tilde I(y) = &\frac34 + \frac{1}{2(Q^2)^2}\left(m_q^4\ln\left|1+\frac{Q^2}{m_q^2}\right| - Q^2 m_q^2\right) \\ &+\int_0^1 dx\left(\frac{-m_q^2}{2q\cdot Qx}\ln\left|\frac{m_q^2+Q^2x(x-1)}{m_q^2+2q\cdot Qx(x-1)}\right|\right. \\ &\qquad +(1-x)\ln\left|\frac{m_q^2+2q\cdot Qx(x-1)}{m_q^2+Q^2}\right| \\ &\qquad \left.+\frac{2q\cdot Qx^2}{\beta}\ln\left|\frac{(1-z_+)(x-z_-)}{(1-z_-)(x-z_+)}\right|\right) \\ &+\frac{i\pi}{2}\Theta(1+\frac{m_q^2}{Q^2})\left[1-\left(\frac{m_q^2}{Q^2}\right)^2\right] - i\pi\Theta(y-y_o)\times \\ &\left[\frac{\sqrt{1-4\alpha}}{2}+\frac{m_q^2}{2q\cdot Q}\ln\frac{x^+}{x^-} - 2q\cdot Q\int_{x^-}^{x^+}dx\,\frac{x^2}{\beta}\right], \end{aligned} \tag{14}$$

[†]The importance of these annihilation diagrams was first realized by Bander *et al.* in Ref. [19] in the context of D meson decays using a nonrelativistic quark model approach.

in which

$$y_o = \epsilon_B + \tfrac{4m_q^2}{M_B^2}, \qquad \alpha = \frac{m_q^2}{M_B^2[y-\epsilon_B]},$$
$$z_\pm = \tfrac{-2q\cdot Qx \pm\beta}{2Q^2}, \qquad x^\pm = \frac{1\pm\sqrt{1-4\alpha}}{2} \tag{15}$$

and $\beta = \sqrt{(2q\cdot Qx)^2 - 4Q^2 m_q^2}$. To obtain this result, it is important to strictly keep only terms that are leading in ϵ_B^{-1}. Subleading terms are higher–twist and require other higher–twist elements (such as transverse momentum degrees of freedom) to maintain gauge–invariance.

Numerical integration yields that

$$\int_0^1 dy\, \tilde\phi_{K^*}(y)\, \tilde I(y) = \begin{cases} .51 + i\,.14 & .71 - i\,.30 \\ .43 + i\,.61 & .64 - i\,.30 \end{cases}, \tag{16}$$

where the results are presented as in Eq. (9). The two columns are the results for the case of the up ($m_q = 0$) and charm quark loops ($m_c = 1.5GeV$) respectively. There is negligible change in the charm quark results taking $1.25GeV < m_c < 1.75GeV$.

III. RESULTS

For the decay $B \to K^*\gamma$, the two relevant amplitudes are $M_{\gamma\mathrm{peng}}$ and $M_{c\bar c\,\mathrm{loop}}$. As indicated by their expressions, in order to reflect the fact that the average virtuality of the exchanged gluon (Q^2) in each of the mechanisms is different, the running coupling and Wilson coefficient are evaluated at scales appropriate to each amplitude. For the dominant piece, $M_{\gamma\mathrm{peng}}$, this occurs at $\mu^2 \approx .5\mathrm{GeV}^2$; $-Q^2$ at the pole in the bottom quark's propagator. In the case of $M_{c\bar c\,\mathrm{loop}}$, $\langle -Q^2\rangle \approx \frac{1}{2}\epsilon_B M_B^2 = 1.3\mathrm{GeV}^2$. Note that a change in scale μ^2 affects the overall magnitude of the branching rate predominantly by the dependence on α_s in Eq. (5).

For $\Lambda_{QCD} = .2$GeV one obtains for the enhancement factor R in the total decay rate the result that

$$R = \frac{(M_{\gamma\mathrm{peng}} + M_{c\bar c\,\mathrm{loop}})^2}{M_{\gamma\mathrm{peng}}^2} = \begin{cases} 1.06 \\ 1.31 \end{cases}, \tag{17}$$

where the two results are for the asymptotic and CZZ distribution amplitudes respectively. Note that in this ratio the uncertainty due to our present ignorance of f_B, as well as the dominant dependence on the parameter ϵ_B cancel. There is little dependence in R on the exact value of Λ_{QCD}. However a precise value for R is clearly dependent upon details of the kaon's soft–physics information (unlike the square modulus of Eq. (9)). The range is due primarily to the phase differences in $M_{\gamma\mathrm{peng}}$ (Eq. (16) shows that $M_{c\bar c\,\mathrm{loop}}$ is nearly insensitive to the choice of $\phi_{K^*}(y)$). One should note that this range is comparable to the various estimates obtained using vector–meson dominance methods [6,7]. Such duality is perhaps best understood by further noting that in pQCD [21] the nonfactorizing amplitudes are found to produce large contributions to the transversally polarized final states of the decay $B \to J/\psi\, K^{(*)}$.

In the case $B^+ \to \rho^+\gamma$, CKM factors no longer suppress either the annihilation diagrams or the up–quark loop contributions. To quantify this dependence, unitarity of CKM matrix is exploited, whereby

$$V_{bu}V_{ud}^* + V_{bc}V_{cd}^* + V_{bt}V_{td}^* = 0. \tag{18}$$

Defining the ratio $V_{bc}V_{cd}^*/V_{bt}V_{td}^* = r$ and assuming SU(3) symmetric distribution amplitudes for the ρ and K^* mesons one obtains for the ratio of decay rates that

$$\frac{\Gamma_{B^+\to\rho^+\gamma}}{\Gamma_{B^+\to K^{*+}\gamma}} = \left(\frac{f_\rho V_{td}}{f_{K^*} V_{ts}}\right)^2 f_{asy,CZZ}(r), \tag{19}$$

where $f_{asy,CZZ}(r) \neq 1$ represents the error due to competing mechanisms of extracting this ratio. The subscripts recall the fact that $f(r)$ is dependent upon the distribution amplitude used for the ρ and K^* mesons. In Fig. (4), $f(r=-2.25)$ and $f(r=-.5)$ are plotted as a function of Λ_{QCD} (the range of r as suggested by the Particle Data Group is $-2.25 \le r \le -.5$, [22]). We observe that while $f(r)$ is fairly independent of the distribution amplitudes, there is significant dependence on Λ_{QCD}. Note that the deviation of $f(r)$ from 1 is greatest for smallest Λ_{QCD}, where one would expect that the perturbative formalism employed here is more accurate. A conservative estimate is therefore that these these decay modes can be used to obtain $|V_{td}/V_{ts}|$ [8] to only within a factor of two or so.

In the case of the neutral B decays, the effective absence of the annihilation diagram improves upon this result. From Fig. (5) we obtain an estimated uncertainty in extracting $|V_{td}/V_{ts}|$ on the order of 50%.

IV. CONCLUSION

The two body exclusive decays of the B meson necessarily involves modeling of the initial and final state hadrons. Perturbative QCD methods have been previously shown [11] to be a robust framework for describing these processes and earlier work [10] had already shown that from the dominant decay mechanism, Fig. (1), the observed rate for $B \to K^*\gamma$ [1] was consistent with a pQCD approach. Using this experience, the question of the relative importance of subdominant processes to the decays $B \to K^*\gamma$ and $B \to \rho\gamma$ have been herein addressed at the leading twist, leading α_s level. By working within a single, coherent framework the possibility of double counting present when using a mixed parton–meson approach has been avoided.

In the case of $B \to K^*\gamma$, subdominant charm quark loops [9], $M_{c\bar{c}\,\rm loop}$, here called double–penguins (and elsewhere in the literature [6,7] as "long–distance", vector meson like states) have been found to yield an enhancement of up to 30% in the total decay rate, depending upon bound–state parameters of the K^*. The uncertainty is due primarily to phase differences between $M_{\gamma\rm peng}$, the dominant mechanism, and $M_{c\bar{c}\,\rm loop}$.

In the case of $B \to \rho\gamma$, which depends upon the unknown CKM matrix elements V_{bu} and V_{td}, additional subdominant processes involving up quark loops and also annihilation diagrams [19] may no longer be CKM suppressed. Assuming $SU_f(3)$ symmetry of the ρ and K^* distribution amplitudes, one can estimate the accuracy to which one could extract [8] $|V_{td}/V_{ts}|$ using the ratio of branching rates $\Gamma_{B^+\to\rho^+\gamma}/\Gamma_{B^+\to K^{*+}\gamma}$. A conservative estimate using present bounds [22] for $V_{bc}V_{cd}^*/V_{bt}V_{td}^*$ and reflecting the sensitivity of this analysis to the various parameters entering the calculation suggests that one can extract the ratio $|V_{td}/V_{ts}|$ to within a factor of two.

ACKNOWLEDGEMENTS

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[1] CLEO collaboration, R. Ammar *et al.*, Phys. Rev. Lett. **71**, 674 (1993). They give branching fraction $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ and see both B^0 and B^-.

[2] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351 (1974); M.A. Shifman, A.I. Vainstein, and V.I. Zakharov, Sov. Phys. JETP **45**, 670 (1977).

[3] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981).

[4] R. Grigjanus *et al.*, Phys. Lett. B **213**, 355 (1988); B. Grinstein, R. Springer, and M.B. Wise, Nucl. Phys. B **339**, 269 (1990); G. Cella *et al.*, Phys. Lett. B **248**, 181 (1990); M. Misiak, Phys. Lett. B **269**, 161 (1991); M. Ciuchini, E. Franco, G. Martinelli, L. Reina, and L. Silvestrini, Phys. Lett. B **316**, 127 (1993); G. Cella, G. Curci, G. Ricciardi, and A. Viceré, Phys. Lett. B **325**, 227 1994; M. Ciuchini, E. Franco, L. Reina, and L. Silvestrini, Nucl. Phys. **B421**, 41 (1994).

[5] CLEO collaboration, M. S. Alam *et al.*, Phys. Rev. **D50**, 43 (1994).

[6] E. Golowich and S. Pakvasa, Phys. Lett. **B205**, 393 (1988); N. G. Deshpande, J. Prampetic and K. Panose, Phys. Lett. **B214**, 467 (1988).

[7] E. Golowich and S. Pakvasa, Phys. Rev. D**51**, 1215 (1995); H. Y. Cheng, "Vector Dominance Effects in Weak radiative Decays of the B Meson", IP–ASTP–23–94, 1994; D. Atwood, B. Blok and A. Soni, "Feasibility of Extracting V_{td} from Radiative $B(B_s)$ Decays", SLAC–PUB–6635, BNL–60709, TECHNION–PH–94–11, 1994. N. G. Deshpande, X. G. He and J. Prampetic, "Long Distance Contributions to Penguin Processes $b \to s\gamma$ and $b \to d\gamma$", OITS–564, 1994.

[8] A. Ali, V. Braun and H. Simma, Z. Phys. **C63**, 437 (1994).

[9] A. Ali and C. Greub, Z. Phys. **C49**, 431 (1991).

[10] C. E. Carlson and J. Milana, Phys. Rev. **D51**, 4950 (1995).

[11] C. E. Carlson and J. Milana, Phys. Rev. D**49**, 5908 (1994).

[12] A. Szczepaniak, E. M. Henley, and S. J. Brodsky, Phys. Lett. **B243**, 287 (1990). In this pioneering work on the subject, some crucial oversights were made which led to a severe underestimate of various branching rates. This result has unfortunately often be quoted to vitiate the applicability of pQCD to these decays.

[13] S. J. Brodsky and G. P. Lepage, Phys. Rev. D**22**, 2157 (1980); *ibid* in "Perturbative QCD" (A. H. Mueller, ed.) (World Scientific, Singapore, 1989).

[14] G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. **43**, 246 (1979); A. V. Efremov and A. V. Radyushkin, Phys. Lett. **B94**, 245 (1980).

[15] V. Chernyak, A. Zhitnitsky, and I. Zhitnitsky, Nucl. Phys. **B204**, 477 (1982).

[16] G. Farrar, G. Sterman, and H. Zhang, Phys. Rev. Lett. **62**, 2229 (1989); R. Kahler and J. Milana, Phys. Rev. D**47**, R3690 (1993).

[17] C. E. Carlson and J. Milana, Phys. Lett. **B301**, 237 (1993).

[18] S. Coleman and R.E. Norton, Nuovo Cim. **28**, 438 (1965).

[19] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **44**, 7 (1980).

[20] C. Greub, H. Simma, and D. Wyler, Nucl. Phys. **B434**, 39 (1995).

[21] C. E. Carlson and J. Milana, "Penguins leaving the Pole: bound state effects in $B \to K^*\gamma$", WM–94–110, U. of MD PP #95–034, (1994).

[22] The Particle Data Group, Phys. Rev. **D50**, 1173 (1994).

FIG. 1. The leading contribution to the decay $B \to K^*\gamma$. The wavy line is a photon, the curly line is a gluon.

FIG. 2. One of the two "double–penguin" contributions to the decay $B \to K^*\gamma$. The second (or photon gluon crossed) graph is required for gauge–invariance and to obtain a ultraviolet finite result.

FIG. 3. The leading annihilation graph. The photon is emitted from the light quark in the B–meson.

FIG. 4. The function $f(r = -2.25)$ (bottom curves) and $f(r = -.5)$ (top curves) for the ratio $\Gamma_{B^+ \to \rho^+\gamma}/\Gamma_{B^+ \to K^{*+}\gamma}$ as a function of Λ_{QCD}. The full lines are the results using the asymptotic distribution amplitude for the vector mesons; the dotted lines are those using the CZZ distribution.

FIG. 5. $f(r = -2.25)$ and $f(r = -.5)$ for $\Gamma_{B^+ \to \rho^0\gamma}/\Gamma_{B^+ \to K^{*0}\gamma}$. Notation same as in Fig. (4).

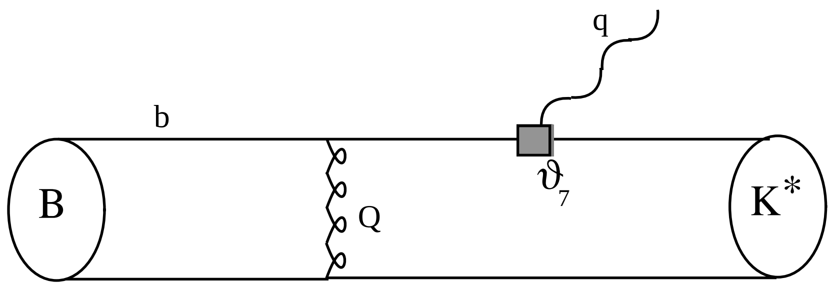


Fig. (1)

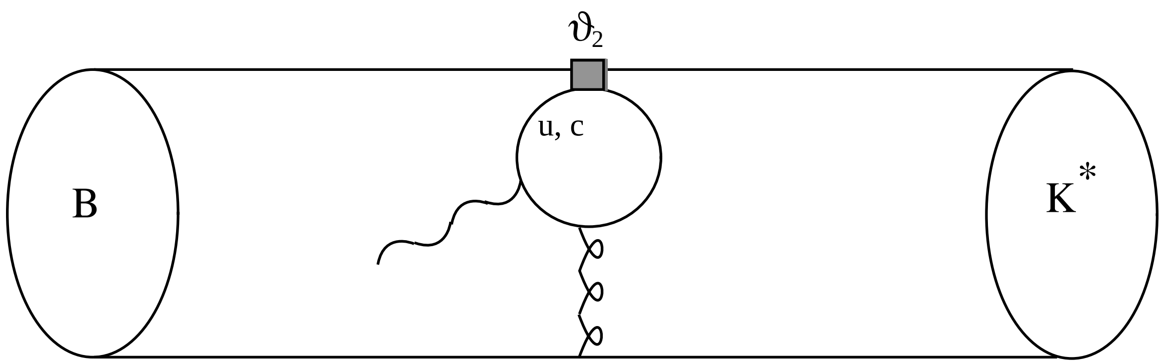


Fig. (2)

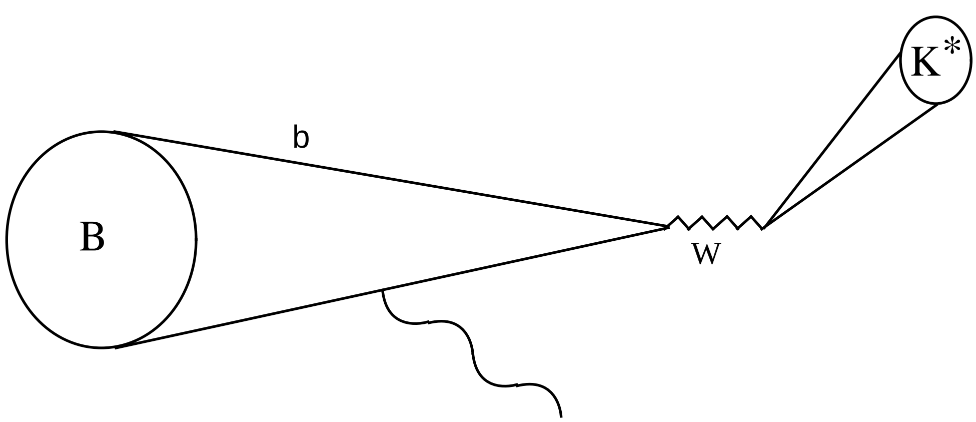
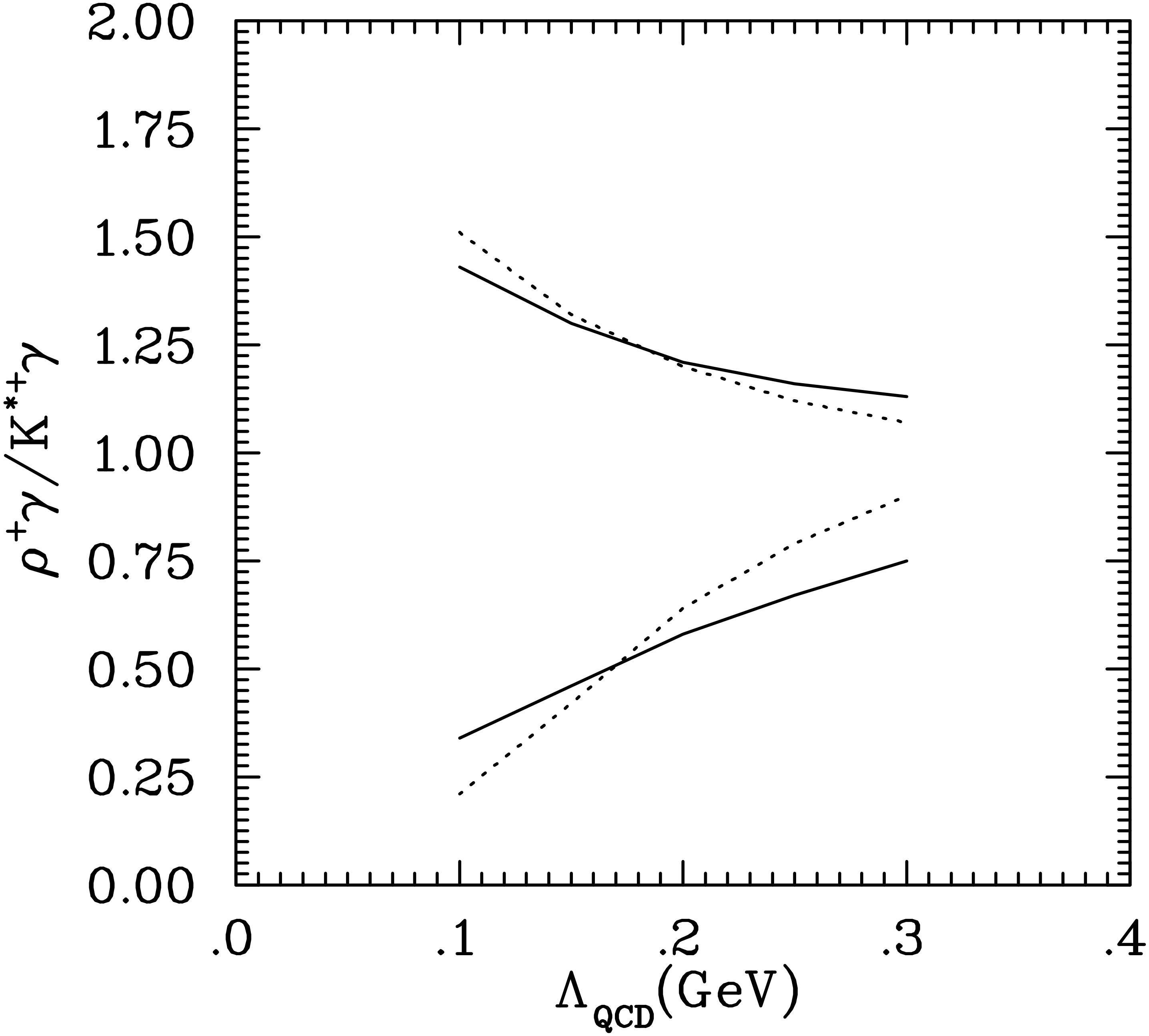
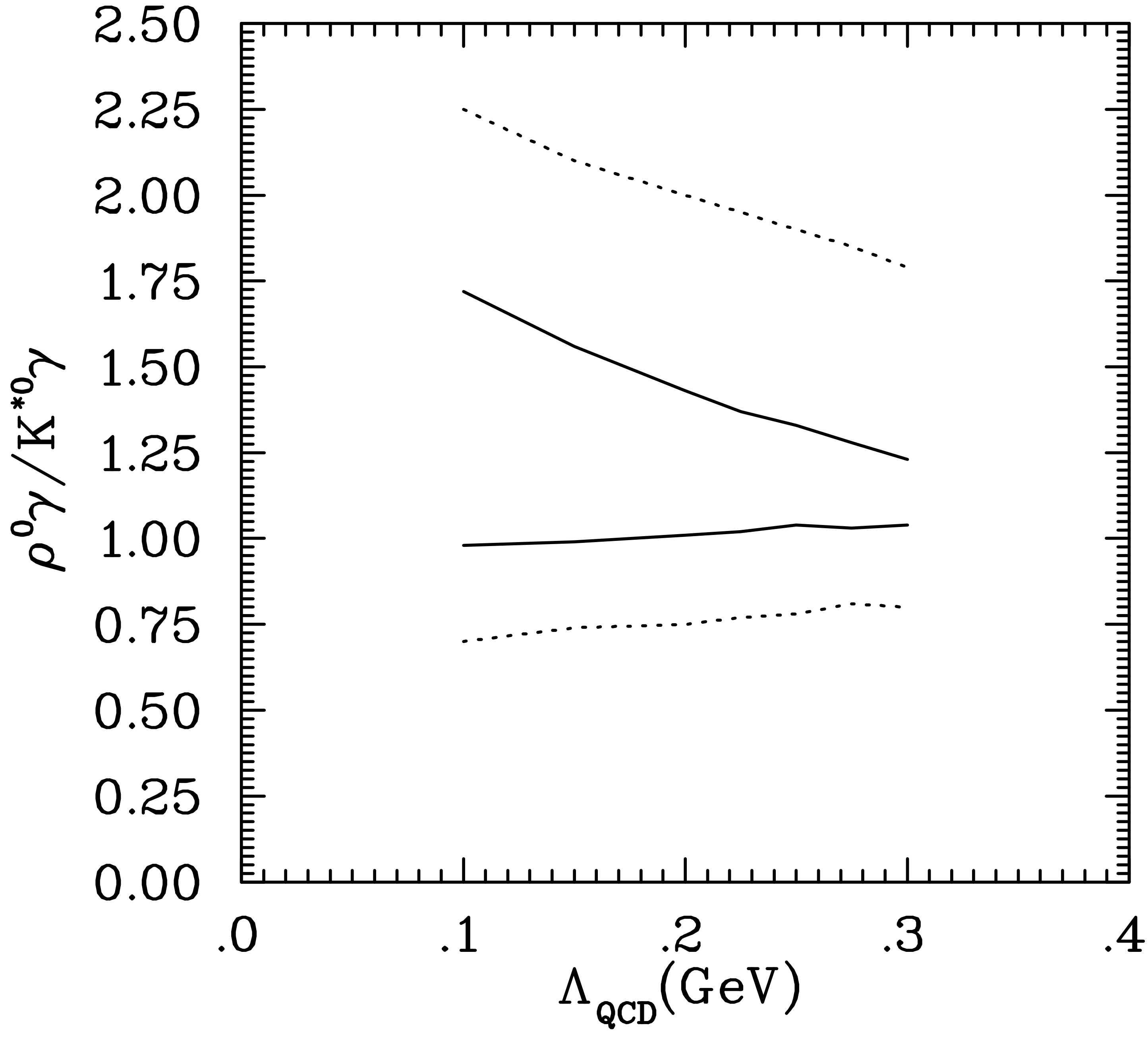


Fig. (3)



2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
.0
.1
.2
.3
.4
$\rho^+\gamma/K^{*+}\gamma$
Λ_{QCD}(GeV)



2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
.0
.1
.2
.3
.4
$\rho^0\gamma/K^{*0}\gamma$
Λ_{QCD}(GeV)